Equilibrium Consequences of Vouchers Under Simultaneous Extensive and Intensive Margins Competition

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Abstract

I develop an empirical model to study the consequences of educational vouchers on markets' equilibria. Unlike most related papers, I model both extensive and intensive margins of action for schools. I apply my model to Chile's primary education, an industry that heavily subsidizes private schools through the use of vouchers. Results show that a correct understanding of the equilibrium consequences of vouchers necessitates accounting for both extensive (program participation) and intensive (fees) margins. I also find that the actual voucher program targeted to vulnerable students attracts mostly low-quality schools to serve low-income families. Alternative policy designs are proposed and tested.

Keywords: Educational vouchers, discrete-continuous games, large games.

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1 Introduction

There is a rich literature in industrial organization (IO) demonstrating the importance of accounting for firms' entry and exit behavior (extensive margin) when evaluating the effects of policies in various industries (Berry and Reiss, 2007; Draganska et al., 2008), which complements the related literature on the consequences of policies on firms' intensive margin strategies, such as price, given a fixed structure or configuration of markets (Nevo, 2000; Ciliberto and Williams, 2014; Miller and Weinberg, 2017). There is also a set of studies that join both strands of the literature, by simultaneously examining firms' responses to policies along both extensive and intensive margins (Draganska et al., 2009; Eizenberg, 2014; Ciliberto et al., 2021). Education markets are not the exception. Studies such as Singleton (2019) and Dinerstein and Smith (2021) analyze schools' entry and exit behavior, whereas Allende et al. (2019) and Bau (2022) investigate schools' responses to policies along continuous measures of quality. However, the task of allowing simultaneous extensive and intensive margins responses to policies has proven difficult in the educational context, given the large number of schools that often characterize education markets.

To understand why solving and estimating games where a large number of players compete in both discrete and continuous strategies is challenging, suppose J players simultaneously choose whether they enter the market and the price they set if they enter. Note that, depending on how large J is, players' entry decisions may lead to a very large number of different counterfactual market configurations, each of which has (at least) one equilibrium profile of prices associated to it. For instance, a market with ten players has 1,024 counterfactual market configurations. Therefore, to solve for equilibria in the entire game, the analyst must solve for price equilibria in each of the 1,024 different market configurations, compute the corresponding players' payoffs (e.g. profits) in each counterfactual configuration, and use the computed payoffs to predict players' entry decisions. This task grows exponentially in difficulty when we increase the number of players. A market with twenty players has 1,048,576 counterfactual market configurations. Thus, considering that urban educational markets are typically comprised by tens, and sometimes hundreds, of schools, solving for equilibria quickly becomes computationally intractable.

In this paper, I fill the above-mentioned gap in the literature by developing and estimating a static game of incomplete information where many schools compete in both discrete and continuous strategies. To overcome the computational difficulties associated to solving discrete-continuous games with many players, I recognize that storing large vectors of counterfactuals associated to many contingencies is not only expensive for the computer, but it also puts a heavy burden on human cognition.[1] Thus, standard concepts of equilibrium (i.e. Bayesian Nash

[1] See a related discussion in Wollmann (2018), that through a series of interviews to managers in vehicle companies argues in favor of the impossibility of firm managers to store large dimensions of contingencies. Graham and Harvey (2001) and Summers (1987) also show evidence that managers often use simple approximations to complicated budgeting

equilibrium), that assume that players are capable to evaluate and contrast thousands or more counterfactual scenarios no longer apply. Other, more realistic, notions of equilibrium that make explicit such limited capability are more appealing in these contexts. In particular, I focus on a well accepted concept of equilibrium for this type of games: *fully cursed* equilibrium (Eyster and Rabin, 2005).

Fully cursed equilibrium recognizes that players have limited capabilities to evaluate counterfactual situations as the number of contingencies grows, in an incomplete information game framework. In this paper's setting, fully cursedness implies that each school correctly predicts the distribution of other schools' actions, but ignores the correlation between these actions and types. This assumption differs with Bayesian Nash equilibrium (BNE), that assumes that each school expects other schools to play according to their type. As a consequence, fully cursedness necessarily adds parsimony and computational tractability to the model.

My empirical application of fully cursed equilibrium adds important flexibility to the theory, motivated by the context, the data, and my own conversations with school principals and managers. More concretely, I allow schools to partially predict their competitors' actions based on a set of observable characteristics. As a result, school heterogeneity is captured by observables, and schools' strong/weak (probable) competitors can be identified. In other words, schools are allowed to take into account the correlation between other schools' actions and the observable part of their type. Fully cursedness is then applied to the unobserved determinants of actions, after controlling for observable characteristics.

I develop an equilibrium model of demand for and supply of schools inspired by Chile's nationwide voucher agenda, that combines two different voucher programs to subsidize enrollment: a universal voucher, which is a per-student subsidy paid to all schools; and a targeted voucher, which is a per-low income student subsidy paid to schools that choose to participate in the targeted voucher program.

The demand side of the model has families choosing schools by taking into account a number of schools' observable (to the econometrician) characteristics, such as proximity, top-up fees, a measure of quality, whether the school is public or private, religious affiliation, as well as unobservable (to the econometrician) characteristics. In the supply side, schools are vertically and horizontally (in characteristics and spatially) differentiated, and compete simultaneously choosing whether they participate in the targeted voucher program, and the fees they charge on top of the subsidies in each participation regime. Though tailored to the Chilean case, my model is general enough to be adapted to other contexts and industries.[2]

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and discounting problems.

[2]See, for instance, Dinerstein et al. (2020), who examine the crowding out effects of public school construction on private school supply in the Dominican Republic, by developing a supply side game inspired by early versions of this paper's model.

My model can be viewed as a multiagent version of the classic (generalized) Roy model (Roy, 1951; Amemiya, 1985; Heckman and Honoré, 1990; Heckman and Vytlacil, 2007). In the generalized Roy model, a decision maker decides between two sectors, and experiments an outcome conditional on the sector that is chosen. A common example is an individual who chooses between a high school diploma and a college degree as her final educational level and, conditional on that choice, she earns a salary on the labor market. Importantly, the fundamental wage functions are generally different across sectors. The decision rule contrasts the potential salary expected to be earned in one sector with the potential salary expected to be earned in the other sector, net of the relative costs associated to choosing the former sector. In my setting, schools choose whether to participate in the targeted voucher program and, conditional on that choice, they set prices and earn profits. They choose to participate in the program if the net (of participation costs) profits associated to participation are greater than the profits associated to non-participation. As in the generalized Roy model, pricing and profits functions are a priori different in the participation and non-participation regimes. In contrast to the generalized Roy model, game-theoretic externalities are present throughout the model.

The estimation of the model poses some important econometric challenges. First, the model may present multiple equilibria. Second, the data include many schools charging zero top-up fees, which are the result of corner solutions in schools' best response functions. Third, since I explicitly model the demand, the model is highly nonlinear, hence computationally expensive. I perform estimation using a constrained optimization GMM-MPEC algorithm, that searches for the unknown parameters minimizing a GMM objective function subject to markets' equilibria.[3] As is known, MPEC algorithms in estimation are robust to multiple equilibria, as markets' equilibria constraints need not hold in early iterations, but only at the optimum. I further implement censored regression models to allow for potential corner solutions in schools' best response functions for top-up fees. Lastly, fully cursedness implemented within an MPEC algorithm ensures computational tractability.

I use the model and its estimated parameters to study the equilibrium consequences of a variety of voucher policies. I first show that a full understanding of the implications of voucher programs necessitates a model that endogenizes schools' both participation and top-up fees strategies. Simulations show that restricted models that allow schools to respond only in one margin, either participation or top-up fees, may largely mispredict equilibrium results. Then, I examine the sorting on school value added induced by the actual targeted voucher program, where mostly low quality schools choose to participate in the program. Similar sorting results have been found

[3]GMM-MPEC stands for Generalized Method of Moments-Mathematical Programming with Equilibrium Constraints. See Dubé et al. (2012) and Su and Judd (2012) for the theory underlying MPEC, and early applications in economics.

for voucher programs in other contexts (Abdulkadiroglu et al., 2018). Such finding motivates the study of alternative, similarly costly, policies that may be better at attracting high quality schools to serve economically disadvantaged students. I find that policies that include a differential voucher, where schools with higher value added receive larger subsidies, are successful in increasing the quality of the pool of schools that participate in the voucher program targeted to low income students. Furthermore, my model offers some degree of fine tuning of the subsidy amounts, which can prove to be useful for policymakers seeking a particular equilibrium result of policies.

This paper builds on important other work. Neilson (2013, 2020) is possibly the most influential study in the recent IO-Education literature, and one of my closest predecessors. Neilson (2020) develops a demand and supply model of school competition to study the competitive and quality distribution consequences of the introduction of the targeted voucher program in Chile. I closely follow Neilson (2013, 2020) in my demand modeling. However, in contrast to Neilson (2020), that uses observed equilibria in the data to draw his conclusions, I explicitly solve and estimate a discrete-continuous game in the supply. This allows me to study a larger set of policy counterfactuals than if I did not solve for equilibria in the supply.

The work in Neilson (2013, 2020) and others motivated a series of papers that use demand for school models to feed supply side games of imperfect competition among schools. For instance, Allende (2019), Allende et al. (2019), and Bau (2022) estimate continuous strategy supply side games where schools compete in quality and/or price to attract students. Similarly, Ferreyra and Kosenok (2018), Singleton (2019), and Dinerstein and Smith (2021) estimate discrete strategy games, where schools choose entry location and/or exit to maximize a profit-based objective function. My model unites these two strands of the literature by allowing schools to simultaneously choose discrete and continuous strategies in a static supply side game.

Static discrete-continuous games have previously been solved and estimated for other industries. See, e.g., Draganska et al. (2009) for ice cream markets, Eizenberg (2014) for the U.S. home PC market, Pakes et al. (2015) for ATMs, Wollmann (2018) for commercial vehicle markets, Fan and Yang (2020) for the U.S. smartphone market, and Ciliberto et al. (2021) and Li et al. (2021) for airline markets. Contrary to these industries, education markets pose a more challenging task in solving and estimating discrete-continuous games, due to the particularly large number of players that play the supply side game.

Finally, this paper's empirical results also contribute to the evidence of voucher programs on various outcomes, much of which is reviewed by Epple et al. (2017).

2 The Educational System and Vouchers in Chile

Since 1981, Chile's primary and secondary educational system operates under a nationwide school choice voucher agenda. Families are unconstrained to choose schools, as long as they are willing to travel the distance to the school and pay the corresponding top-up fees, which are often zero or very low. Schools, on the other hand, depending on their management type, are subsidized by the government through per student vouchers.

Schools can be grouped into three types, according to their ownership, management, and whether they receive subsidies. *Public* schools are owned and managed by local municipalities, are funded by the voucher subsidies, and are tuition free. *Private-voucher* schools are privately owned and managed, receive subsidies, and are allowed to charge fees to parents on top of the vouchers. About 53% of these schools charged positive top-up fees in 2013, which were on average ∼\$500 (annual). The third group of schools are *private-non-voucher* schools, which are privately owned and managed, do not receive subsidies, and are financed entirely through the fees they charge to parents. On average, these schools charged ∼\$6,400 in 2013.

The vast majority of primary education students attend a subsidized school. In 2013, 40% of primary education students attended a public school, 52% attended a private-voucher school, and only 7% were enrolled in a private-non-voucher school. These figures highlight 1) the important presence of private education in the country, especially of subsidized private schools, and 2) the wide reach of vouchers, where more than 90% of students use them to attend a public or a subsidized private school.

Important for my model and empirical approach is how vouchers are allocated among providers. The government subsidizes schools via two different types of vouchers. The first one and the oldest in the system is what I call the *universal* voucher. This subsidy is a flat amount of funds transferred to all public and private-voucher schools, depending on the number of students enrolled in the school. In 2013, the value of the universal voucher was about \$1,305 per student per year.[4] The second type of subsidy is the *targeted* voucher. This voucher was introduced in the system only in 2008, as an effort to inject extra funds to schools that enroll vulnerable students, who are presumably more costly to educate (Fontaine and Urzúa, 2018). Another motivation for this voucher was to expand the school choice set of low income students, who were thought to be budget constrained when considering enrolling in the most expensive subsidized private schools. As a consequence, the targeted voucher required schools to charge zero top-up fees to eligible low income students. A distinctive feature of this program is that subsidized schools are not forced to participate in it, but are rather invited to join. Schools that opt in receive extra funds through the targeted voucher for every low income student they enroll, with the requirement of charging

[4]This amount slightly varies by grade and by urban status of the school.

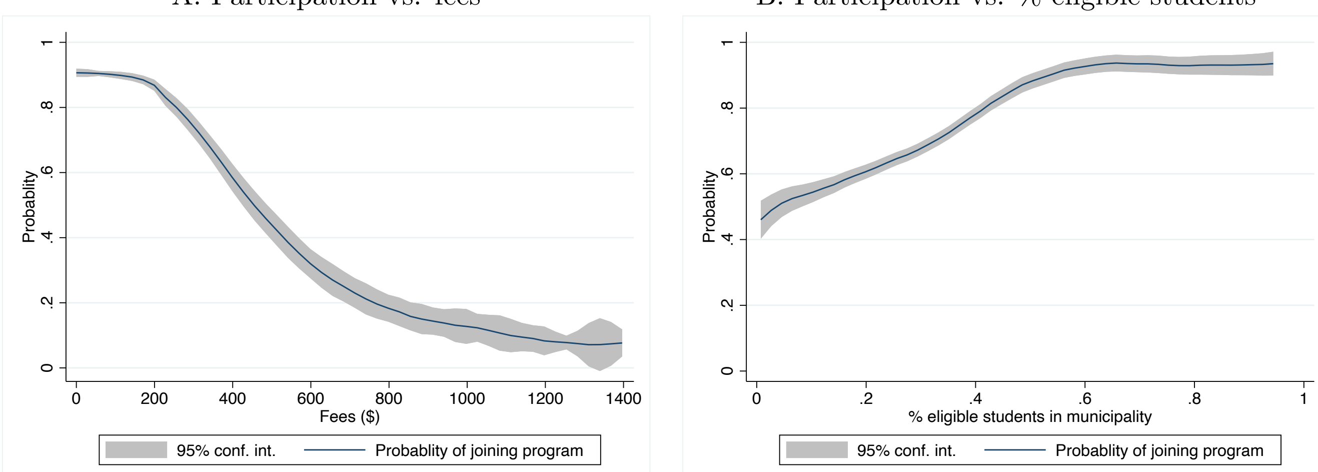
zero top-up fees to these students. Participating schools are still allowed to charge any (possibly positive) top-up fees to higher income students, but do not receive the targeted subsidy for these non-vulnerable students. Schools that decide not to join the targeted program continue to receive the flat universal voucher and to charge top-up fees (if they wish to do so). By 2013, virtually all public schools had joined the targeted program, and ~72% of private-voucher schools had opted in. The value of the targeted subsidy was about $862 that year.

Student eligibility for the targeted voucher is mostly based on family income. Children from families in the first tercile of the income distribution are eligible to receive the targeted voucher, as well as children from families that are beneficiaries of the *Chile Solidario* social program. In 2013, 52% of primary education students were considered eligible for the targeted voucher program.

A final note on the context is that private-voucher schools' decision to participate in the targeted voucher program is unsurprisingly not random. Moreover, schools' participation decisions are very correlated with the fees they charge and with the local share of eligible low income students. As Figure 1 shows, low-fee schools are very likely to participate in the program, while higher-fee schools are less likely to join, in 2013 (panel A). Similarly, schools located in areas with a high share of eligible students join more often than schools in areas with only few of these students (panel B).

Figure 1: Private-Voucher Schools' Program Participation

A. Participation vs. fees B. Participation vs. % eligible students



Notes: This figure displays a nonparametric estimate of the relationship between private-voucher schools' participation in the targeted program and the top-up fees they charge to families (panel A), as well as a nonparametric estimate of the relationship between private-voucher schools' participation in the targeted voucher program and the share of eligible low income students in the schools's municipality (panel B). Data are for the year 2013.

Such participation behavior is in line with schools maximizing profit or having an objective function that heavily depends on profits. To make it clear, think of the revenues of a subsidized school that does not participate in the targeted program. For each student, the school receives the universal voucher plus the fees it charges to parents. Now, if the school participated in the program, it would not change its revenue from non-vulnerable students, but for each low income student it would receive the universal voucher plus the targeted voucher, and nothing from fees. Thus, a profit maximizing school faces the tradeoff of not joining the program and charging fees to low income students versus joining the program and receiving the targeted voucher for these students. Consequently, low-fee schools find it profitable to join the program, while the opposite occurs with high-fee schools. This effect becomes more important where a large share of school's demand comes from low income students.

3 The Model

I develop a structural model of demand for and supply of schools motivated by the voucher programs for primary education in Chile. The model is static. There exist several education markets that are geographically separated one from another. Each market is populated by households that live in different locations within the market, with one child that attends primary education. Given its budget constraint, each household chooses among the schools available in the market.

Schools are (exogenously) distributed within the market's area. There are three different types of schools: public, private-voucher, and private-non-voucher. Public schools are tuition free, while both private-voucher and private-non-voucher schools are allowed to charge fees. Public and private-voucher schools receive a per-student flat subsidy for every student that they enroll, the universal voucher. In addition, a complementary subsidy program is available for public and private-voucher schools: a targeted voucher to economically disadvantaged students. Participation in this targeted program is optional for schools. The targeted voucher program adds extra per-pupil funds over the universal voucher for every eligible low income student that the school enrolls, with the requirement of charging zero fees to those students. Participating private-voucher schools can still, however, charge a non-zero fee on top of the universal voucher to higher income students. Non-participating private-voucher schools receive the universal voucher and their fees for every student. Private-non-voucher schools do not receive any subsidy, and charge a uniform level of fees to all students.

3.1 Demand

Students have heterogeneous preferences over schools' fees, quality (i.e. a measure of how much the school increases students' test scores), geographical proximity, a set of schools' observable (to the econometrician) characteristics, and schools' unobservable (to the econometrician) characteristics. I capture heterogeneity in preferences with a set of coefficients that vary over students' observed demographic characteristics. Formally, in each market $m \in \{1, \ldots, M\}$, student $i \in \{1, \ldots, I\}$ chooses the school $j \in \{1, \ldots, J\}$ that maximizes her utility. I specify the student's conditional indirect utility by:[5]

$$U_{ij} \quad = \quad \beta_{1i} p_j^{\zeta} + \beta_2^{\zeta} d_{ij} + \beta_{3i}' X_j^{\beta} + \beta_4^{\zeta} q_j + \xi_j^{\zeta} + \varepsilon_{ij} \tag{1}$$

where the superscript $\zeta \in \{L, H\}$ refers to the eligibility status of the student, i.e. low income or high income. Thus, p_j^{ζ} is school j's fees charged to students of type ζ, d_{ij} is distance from student i's home to school j, X_j is a vector of school j's characteristics, q_j is school j's quality measure, ξ_j^{ζ} is the common preference that students of type ζ have for school j's unobservable (to the econometrician) characteristics, and ε_{ij} is an i.i.d. preference shock. Also, for any $\beta^{\zeta} \in \{\beta_2^{\zeta}, \beta_4^{\zeta}, \xi_j^{\zeta}\}$, let $\beta^{\zeta} = D_i \beta^L + (1 - D_i)\beta^H$, where $D_i = \mathbb{1}[i \text{ is low income}]$. Similarly, for $k = 1, 3$, $\beta_{ki} = D_i \beta_{ki}^L + (1 - D_i)\beta_{ki}^H$, where $\beta_{ki}^L = \beta_k^L + \sum_r z_{ir} \beta_{kr}^L$ and $\beta_{ki}^H = \beta_k^H + \sum_r z_{ir} \beta_{kr}^H$, with z_{ir} being student i's demographic characteristic r.

Note that the fees that school j charges to student i, p_j^{ζ}, depend on whether the student is low income, and on whether the school participates in the targeted voucher program. Specifically,

$$p_j^{\zeta} = \tau_j (1 - D_i) p_j^1 + (1 - \tau_j) p_j^0,$$

where $\tau_j = \mathbb{1}[j \text{ participates in the targeted program}]$, p_j^1 is school j's counterfactual fees in the case the school participates in the targeted program, and p_j^0 is school j's counterfactual fees in the case the school does not participate in the targeted program.

Let $V_{ij} = \beta_{1i} p_j^{\zeta} + \beta_2^{\zeta} d_{ij} + \beta_{3i}' X_j + \beta_4^{\zeta} q_j + \xi_j^{\zeta}$. Thus, $U_{ij} = V_{ij} + \varepsilon_{ij}$. Assuming $\varepsilon_{ij} \sim$ Type I Extreme Value, the probability that student i chooses school j is logistic:

$$P_{ij} \quad = \quad \frac{e^{V_{ij}}}{\sum_k e^{V_{ik}}}.$$

[5] I suppress the market subscript m for ease of exposition.

3.2 Supply

I model strategic decisions for private-voucher schools only. I do so because in the data most of the variation needed for econometric identification is available only for private-voucher schools. For instance, the data show that virtually all public schools join the targeted program, which does not provide any variation to identify the participation margin for these schools. Other related studies adopt similar modeling decisions (see, e.g., Allende et al., 2019).[6]

I assume private-voucher schools make their strategic decisions in the context of a static game of incomplete information. Let $a_j = (\tau_j, p_j^1, p_j^0)$ denote the actions available to school j, where $\tau_j \in \{0, 1\}$ is school j's decision to participate in the targeted program, $p_j^1 \geq 0$ is the price the school sets if it joins the program, and $p_j^0 \geq 0$ is the price it sets if it does not join the program. Also, let κ_j be school j's any motive other than revenues that the school weighs in its participation decision, including willingness to serve vulnerable students and additional fixed costs due to increased bureaucracy and monitoring from the government. In addition, let $c_j^1 = (c_j^{1,H}, c_j^{1,L})$ and c_j^0 be school j's marginal costs of educating a student in the in-the-program and in the not-in-the-program regimes, respectively. The term $c_j^{1,H}$ is the marginal cost of educating a high income student, and the term $c_j^{1,L}$ is the marginal cost of educating a low income student, in the case the school joins the targeted program. Collect κ_j and the costs terms in $t_j = (\kappa_j, c_j^1, c_j^0)$. I assume t_j is private information. Finally, denote school j's pure strategy as $s_j(t_j) = \Big(\tau_j(t_j), p_j^1(t_j), p_j^0(t_j)\Big)$.

Private-voucher school j of type t_j chooses its actions to maximize an objective function that combines expected profits and the nonprofit/fixed cost κ_j term. Specifically,

$$\max_{\tau_j \in \{0,1\}, p_j^1 \geq 0, p_j^0 \geq 0} \quad E_{t_{-j}} \left[\tau_j \left(\Pi_j^1(a_j, s_{-j}(t_{-j}); t) + \kappa_j \right) + (1 - \tau_j) \Pi_j^0(a_j, s_{-j}(t_{-j}); t) \right],$$

where,

$$\begin{aligned}
\Pi_j^1(a_j, s_{-j}(t_{-j}); t) \quad &= \quad (p_j^1 + v^u - c_j^{1,H}) \sum_i (1 - D_i) P_{ij}(a_j, s_{-j}(t_{-j}); t) \\
&\quad + (v^u + v^t - c_j^{1,L}) \sum_i D_i P_{ij}(a_j, s_{-j}(t_{-j}); t), \\
\Pi_j^0(a_j, s_{-j}(t_{-j}); t) \quad &= \quad (p_j^0 + v^u - c_j^0) \sum_i P_{ij}(a_j, s_{-j}(t_{-j}); t).
\end{aligned}$$

The expression for school j's objective function consists of two parts. The first part, $(\Pi_j^1 + \kappa_j)$, is the sum of profits and nonprofit/fixed cost motives associated to joining the program. In that

[6] Other simplifications are also present in the supply side model. See appendix A for a discussion on the most relevant modeling assumptions.

regime, for each high income student, the school receives the fees it charges, p_j^1, the universal voucher, v^u, and it incurs in a marginal cost $c_j^{1,H}$. For each low income student, the school receives the universal and targeted vouchers, $v^u + v^t$, and it incurs in a marginal cost $c_j^{1,L}$. The school also internalizes a combination of nonprofit gains and participation costs, summarized in κ_j. The second part of the school j's objective function, Π_j^0, is the profits associated to not joining the targeted program. In such case, for each student, regardless of her socioeconomic status, the school receives the fees it charges, p_j^0, the universal voucher, v^u, and it incurs in a marginal cost, c_j^0.[7]

The standard equilibrium notion for this type of games is Bayesian Nash equilibrium. Such equilibrium concept assumes that schools do not observe the types of their opponents but know the probability distribution from which each type is drawn from. Moreover, schools expect that their opponents choose their actions according to their types. That is, schools anticipate a correlation between other schools' actions and types.

In large discrete-continuous games, BNE involves the computation of many counterfactual equilibria in the continuous strategies to form the expected utility function of each player. This procedure quickly becomes expensive, computationally, but also in terms of players' cognition.

For illustration, note that in a BNE framework, school j's belief of the expected profits in the not-in-the-program regime is,

$$E_{t_{-j}}^{BN}\left[\Pi_j^0(a_j, s_{-j}(t_{-j}); t)\right] = \int_{\mathcal{S}_{t_{-j}}} (p_j^0 + v^u - c_j^0) \sum_i P_{ij}(a_j, s_{-j}(t_{-j}); t) dF(t_{-j}|t_j),$$

where $\mathcal{S}_{t_{-j}}$ is the space of other schools' types, and $F(t_{-j}|t_j)$ is the conditional probability distribution of other schools' types. Using iterated expectations, school j's expected profits in the not-in-the-program regime can be rewritten as,

$$E_{t_{-j}}^{BN}\left[\Pi_j^0(a_j, s_{-j}(t_{-j}); t)\right] = \sum_{\tau_{-j}} \left[\int_{\mathcal{S}_{c_{-j}}} (p_j^0 + v^u - c_j^0) \sum_i P_{ij}(a_j, s_{-j}(t_{-j}); t) dG(c_{-j}|\tau_{-j}; t_j)\right] Pr(\tau_{-j}|t_j),$$

where $\mathcal{S}_{c_{-j}}$ is the space of other schools' marginal costs, and $G(c_{-j}|\tau_{-j}; t_j)$ is the conditional probability distribution of other schools' marginal costs. The term inside the square brackets is the conditional expected profits in the not-in-the-program regime for a particular value of τ_{-j} (i.e. other schools' program participation decisions), and the summation takes the expectation of those profits over all possible values of τ_{-j} (i.e. market configurations). Note that,

[7] Notice that I assume different marginal costs for low and high income students in the participation regime, and the same marginal cost for all students in the non-participation regime. Although it may seem more natural to assume different marginal costs for low and high income students also in the non-participation regime, the empirical model does not identify these costs separately.

for a given τ_{-j}, conditional expected profits are a function of equilibrium counterfactual prices, $\big(p^1(\tau_j = 0, \tau_{-j}), p^0(\tau_j = 0, \tau_{-j})\big)$. Thus, to compute the expected profits in the not-in-the-program regime, we first need to obtain the equilibrium prices and the corresponding conditional expected profits for each τ_{-j}. Then, we take the weighted sum of all conditional expected profits, where the weights are the probabilities of occurrence of each τ_{-j}. This is a complicated task for large games. For instance, in a market with $J - 1 = 10$, 1,024 different sets of equilibrium prices need to be obtained. In a market with $J - 1 = 20$, 1,048,576 different sets of equilibrium prices need to be obtained. And in a market more typical of the setting I study in this paper, with $J - 1 = 40$, $5.498e^{11}$ different sets of equilibrium prices need to be obtained.[8] This procedure becomes computationally intractable very fast.

I address the computational intractability related to BNE by recognizing that it is unrealistic to assume that schools can store large dimensions of counterfactuals to form their best responses.[9] In fact, there is a large empirical literature showing that people do not understand the strategy of their opponents state by state.[10] As a consequence, standard equilibrium concepts, that assume that players are perfectly rational in their ability to form correct expectations about other players' behavior and to select best responses to these expectations no longer apply.

I therefore depart from the standard Bayesian Nash equilibrium framework, and instead assume that schools are fully cursed (Eyster and Rabin, 2005).[11] As a first step to defining fully cursed equilibrium in this game, let $\bar{s}_{-j}(t_j) \equiv \int_{\mathcal{S}_{t_{-j}}} s_{-j}(t_{-j}) dF(t_{-j}|t_j)$. That is, $\bar{s}_{-j}(t_j)$ is the average strategy of other players, averaged over other players' types, from the perspective of school j of type t_j.

When schools are fully cursed, they (mistakenly) believe that each type profile of the other players plays the same action profile, $\bar{s}_{-j}(t_j)$, whenever they play $s_{-j}(t_{-j})$. Consequently, fully

[8] In general, 2^{J-1} equilibrium prices and associated profits need to be computed.

[9] See the related discussion in Jehiel (2005) about the impossibility of a chess player to know what the opponent might do at every board position.

[10] See, e.g., Kagel and Levin (1986, 2002), Thaler (1988), Holt and Sherman (1994), Converse (2000), Charness and Levin (2009), Esponda and Vespa (2014).

[11] In section 3.3 below, I revise the theoretical and empirical motivations for fully cursed equilibrium, and lay out arguments supporting the choice of this equilibrium concept.

cursed school j's beliefs of the expected profits in each regime are such that,

$$\begin{aligned}
E^{FC}_{t_{-j}}\left[\Pi^1_j(a_j, s_{-j}(t_{-j}); t)\right] &= \Pi^1_j(a_j, \bar{s}_{-j}(t_j); t_j) \\
&= (p^1_j + v^u - c^{1,H}_j)\sum_i (1-D_i)P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j) \\
&\quad +(v^u + v^t - c^{1,L}_j)\sum_i D_i P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j), \\
E^{FC}_{t_{-j}}\left[\Pi^0_j(a_j, s_{-j}(t_{-j}); t)\right] &= \Pi^0_j(a_j, \bar{s}_{-j}(t_j); t_j) \\
&= (p^0_j + v^u - c^0_j)\sum_i P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j).
\end{aligned}$$

Define $\Pi_j(a_j, \bar{s}_{-j}(t_j); t_j) \equiv \tau_j \left(\Pi^1_j(a_j, \bar{s}_{-j}(t_j); t_j) + \kappa_j\right) + (1-\tau_j)\Pi^0_j(a_j, \bar{s}_{-j}(t_j); t_j)$, i.e. the objective function for fully cursed school j of type t_j.

Definition 3.1 *In the static game of incomplete information described above, a pure strategy profile $\bar{s}^* = (\bar{s}^*_1 \ldots, \bar{s}^*_J)$ is a* ***fully cursed equilibrium*** *if and only if for each j and for each t_j,*

$$a^*_j \in \arg\max_{a_j} \Pi_j(a_j, \bar{s}^*_{-j}(t_j); t_j).$$

In a fully cursed equilibrium, each school correctly predicts the probability distribution over its opponents' actions, but ignores the correlation between other schools' actions and their types. As a consequence, fully cursed equilibrium does not suffer from the curse of dimensionality problem present in BNE, since only one set of equilibrium prices (and program participation decisions) needs to be computed, regardless of the number of opponents.

When schools' types are independent—meaning that for each t_j, t'_j, t_{-j}, $F(t_{-j}|t_j) = F(t_{-j}|t'_j)$—then each type of player j and any type of player k share common beliefs about the strategy of any player $l \neq j, k$; namely, $\bar{s}_l(t_j) = \bar{s}_l(t'_j) = \bar{s}_l(t_k) = \bar{s}_l(t'_k)$, for any t_j, t'_j, t_k, t'_k.

School j's best-response functions satisfy,

$$\begin{aligned}
p_j^1(a_j, \bar{s}_{-j}(t_j); t_j) &\geq c_j^{1,H} - v^u - \frac{\sum_i (1 - D_i) P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j)}{\sum_i (1 - D_i) \frac{\partial P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j)}{\partial p_j^1}}, & (2) \\
0 &= p_j^1(a_j, \bar{s}_{-j}(t_j); t_j) \frac{\partial \Pi_j^1(a_j, \bar{s}_{-j}(t_j); t_j)}{\partial p_j^1}, & (3) \\
p_j^1(a_j, \bar{s}_{-j}(t_j); t_j) &\geq 0, & (4) \\
p_j^0(a_j, \bar{s}_{-j}(t_j); t_j) &\geq c_j^0 - v^u - \frac{\sum_i P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j)}{\sum_i \frac{\partial P_{ij}(a_j, \bar{s}_{-j}(t_j); t_j)}{\partial p_j^0}}, & (5) \\
0 &= p_j^0(a_j, \bar{s}_{-j}(t_j); t_j) \frac{\partial \Pi_j^0(a_j, \bar{s}_{-j}(t_j); t_j)}{\partial p_j^0}, & (6) \\
p_j^0(a_j, \bar{s}_{-j}(t_j); t_j) &\geq 0, & (7) \\
\tau_j(a_j, \bar{s}_{-j}(t_j); t_j) &= \mathbb{1}\left\{ \left(\Pi_j^1(a_j, \bar{s}_{-j}(t_j); t_j) + \kappa_j \right) - \Pi_j^0(a_j, \bar{s}_{-j}(t_j); t_j) > 0 \right\}, & (8)
\end{aligned}$$

where equations (2) and (5) are the first order conditions for p_j^1 and p_j^0, respectively, equations (3) and (6) are the corresponding complementary slackness conditions, and equations (4) and (7) are the corresponding non-negativity constraints. Note that p_j^1 is a function of the marginal cost of educating a high-income student in the in-the-program regime, $c_j^{1,H}$, the universal voucher, v^u, and the markup the school charges over the price under perfect competition with subsidy, which is the last term on the right-hand side of equation (2). This markup term depends on the price-elasticity of the demand of high income students, and not on that of low income students, since it is only high income students that face this price. Also, the markup is lower the more price-elastic is the demand, as is usual in imperfect competitive markets. Note, too, that the universal voucher enters equation (2) linearly, and therefore acts as a marginal cost reducer. Similarly, p_j^0 depends on the marginal cost of educating a student in the not-in-the-program regime, the universal voucher, and a markup term. Here, the marginal cost c_j^0 does not vary with the socioeconomic status of the student, and the markup term depends on the price-elasticity of the demand of all (low and high income) students, because all students face this price. Equation (8) is the optimality condition for τ_j, and states that school j joins the targeted program if and only if the combination of profits and non-profit/fixed cost motives associated to joining the program are greater than the profits associated to not joining the program.

As stated in definition 3.1, a fully cursed equilibrium in this game is the fixed point of all schools' first order conditions.

3.3 Theoretical and Empirical Motivations for Fully Cursed Equilibrium

I adopt Eyster and Rabin (2005)'s concept of fully cursed equilibrium.[12] Fully cursed agents have correct beliefs about the distribution of others' types, but fail to account for the correlation between other players' types and actions. Fully cursed agents, therefore, best respond to their opponents' average distribution of actions. Fully cursed equilibrium is a limit case of Eyster and Rabin (2005)'s more general concept of (partial) cursed equilibrium, that assumes that each player plays a best response to a convex combination of others' actual strategies and the aggregate distribution. A disadvantage of this intermediate case of cursed equilibrium is that it is hard to imagine a learning process that leads to such concept (Eyster and Rabin, 2005; Fudenberg, 2006). On the contrary, fully cursed equilibrium is founded on a well defined learning process, in which agents observe others' actions but neither their types nor their payoffs.

Fully cursedness is related to other equilibrium concepts of games with agents that fail to account for the informational content of others' play. Self-confirming equilibrium (Fudenberg and Levine, 1993; Dekel et al., 2004; Battigalli et al., 2015) does not assume that players have correct beliefs about the distribution of opponents' play, but only that the beliefs are consistent with what players observe when the game is played—i.e. beliefs are correct only along the equilibrium path. Fully cursed equilibrium corresponds to a self-confirming equilibrium where players only observe the aggregate distribution of others' actions but not their types. Relatedly, Esponda (2008) combines self-confirming equilibrium with some monotonicity restrictions to study the impact of naive agents on equilibrium play in adverse selection models. Jehiel and Koessler (2008)'s analogy-based expectation equilibrium (ABEE) assumes that players bundle states into analogy classes and best respond to opponents' average strategy in those analogy classes.[13] Analogy classes simply are a coarse partition of the distribution of states and types. Fully cursed equilibrium is a special case of ABEE, in which all states are bundled into a common analogy class. Moreover, ABEE can be viewed as a natural selection of self-confirming equilibrium, in which the signals received by players after each round of play correspond to the average play in each analogy class. Finally, Esponda and Pouzo (2016)'s Berk-Nash equilibrium presents a unifying framework of equilibrium models of games with agents with misspecified views of their environment, that includes Bayesian Nash equilibrium, self-confirming equilibrium, ABEE, and fully cursed equilibrium as special cases.

All of the above-mentioned theories constitute a response to a body of evidence on bounded rationality that is both economically significant and regular enough to be modeled (Fudenberg,

[12] For early developments of the concept of fully cursed equilibrium, see Kagel and Levin (1986), and Holt and Sherman (1994).

[13] See, also, Jehiel (2005).

2006). The winner's curse is early documented in experimental and non-experimental studies of auctions and trade with adverse selection (Kagel and Levin, 1986, 2002; Thaler, 1988; Holt and Sherman, 1994; Charness and Levin, 2009).[14] Individuals' failure to account for the informational content of other people's actions is also empirically present in contexts of voting in elections and juries (Converse, 2000; Esponda and Vespa, 2014). More importantly, Kagel and Levin (1986), and Eyster and Rabin (2005) find that these phenomena are more likely to arise in large games, highlighting the limits of human cognition and the difficulty of evaluating counterfactual situations as the number of players/types/states grows—similar to the storage and memory limits of a personal computer.[15]

To complement the theoretical and empirical arguments for fully cursed equilibrium just stated, I interviewed a sample of managers and principals in private-voucher schools in different markets across the country.[16] The following relevant ideas were common in all conversations. Managers explicitly stated that they do not identify a single school or group of schools as direct competitors, i.e. schools whose actions they closely monitor when setting their own strategies. In contrast, they make their decisions looking at the market as a whole. In fact, some managers suggest that they strategically follow a sort of market level summary statistic on the actions of their competitors. This argument rules out models where schools play BNE against a group of close opponents, and are less attentive to the rest of the players—similar to the play in (partial) cursed equilibrium (Eyster and Rabin, 2005). Relatedly, managers know the market well, they know who are the other schools, their religious status, the educational focus and grades they offer, and where in the market's space they are located. In the econometrician jargon, they observe the observable characteristics of the other schools. This is consistent with schools being fully cursed conditional on a set of observed determinants of strategies. Lastly, school managers are well informed about their demand, including the residential location and price sensitivity of families.

In sum, fully cursed equilibrium is a well accepted equilibrium model of games with agents that fail to account for the informational content of other players' actions. It is a special case of several other related models, and is founded on a realistic learning process. It is parsimonious, and therefore computationally tractable. More importantly, fully cursed equilibrium approximates the play of agents in this paper's setting very well.

[14] The winner's curse is generally defined as the winner's disappointment after the auction or trade takes place (Thaler, 1988).

[15] See related discussions in Jehiel (2005), Jehiel and Samet (2007), Jehiel and Koessler (2008), and Esponda and Vespa (2014).

[16] I talked to managers and principals from Colegio Inmaculada Concepcion at Puerto Varas, Colegio San Francisco Javier at Cerro Navia, Colegio Augusto Winter at Temuco, Colegio Santa Marta at La Union, Colegio San Miguel at Calbuco, Colegio Sagrada Familia at Hornopiren, Liceo San Conrado at Futrono, and Colegio Maria Deogracia at Futrono.

4 Data and Educational Markets

I combine various administrative data sets for Chilean schools and students for the year 2013. First, I use the registry of all operating schools, in which I observe schools' management type, fees, subsidies, participation in the targeted voucher program, geocoded address, and other characteristics such as religious orientation and urban status. Second, I use the registry of all students attending primary education in the country. I observe students' grade and school of attendance, eligibility for the targeted program, (non-geocoded) residential address, and other characteristics such as gender and date of birth. Third, I use records on students' performance in mandatory standardized tests taken by all 4th graders in the country. Fourth, the standardized exams include a questionnaire to parents that provide data on demographic characteristics such as parents' level of education, household income, and house amenities (e.g. computer and internet availability), which I also use.[17]

The administrative records provide geocoded addresses for schools, but only name addresses for students' residences. I use a combination of GIS tools to obtain geographic coordinates from students' data, to then calculate home-to-school distance values. This process is key in order to specify a sensitive demand and supply model, because, as I show below, geographical proximity is a strong determinant of school choice and competition.

I also collect data on private-non-voucher schools' fees. Such information is not included in the administrative data from the Ministry of Education. I perform this process by manually collecting fee amounts from schools' websites and telephone conversations. I successfully retrieve fee values for all private-non-voucher schools in the country.

An intermediate step to construct the final sample is to define educational markets. Such task is challenging in the context of urban centers in Chile, as is the case in other Latin American countries (Neilson, 2013; Allende, 2019; Dinerstein et al., 2020). Unrestricted choice combined with a large supply of public and private schools make it difficult to draw clear boundaries of school markets. As illustration, a student that lives close to the border of a city may be attracted to attend a school located in a nearby different city. To tackle this challenge, I develop an algorithm inspired by the work in Neilson (2013), that constructs markets based on student enrollment data. I start with all students in a municipality. I join the starting municipality with all municipalities where the schools chosen by at least 5% of the students are located in. If all students attend schools located in the same municipality, I stop. If students attend schools in other municipalities, then I take all students in the original and these other municipalities and redo the exercise. This creates a network of municipalities that constitutes a market.

I select large educational markets for my final sample. This selection criterium is necessary to

[17]See appendix B for a more detailed description of the administrative data sets I use in this paper.

bring my model and its supply side game with many players to the data. Specifically, I include markets with 10,000 or more primary education (grades 1st–8th) students. In addition, and only to work with markets that are more similar in size to each other, I leave the capital city, Santiago, out of the final sample. Including Santiago is straightforward and does not change the policy recommendation results. I end up with 27 markets, that enroll ∼44% of the student population, and include ∼41% of all schools.[18] Lastly, I keep only 4th graders in the estimation sample, since data on standardized test scores are only available for these students.[19]

A few market descriptive statistics summarized in Table 1 are worth mentioning. The average market has 2,876 4th grade students, of which 54% are eligible for the targeted voucher. The largest market has 7,116 4th grade students. There are 77 schools in the average market, 37 of which are public, 35 are private-voucher, and 4 are private-non-voucher. The largest market has 105 private-voucher schools. Among all private-voucher schools in the average market, about two thirds participate in the targeted voucher program. These figures underscore the (potentially) wide reach of the targeted voucher among students and schools, as well as the large size of the game that is played by private-voucher schools in the supply, which makes my model suitable for the context.

Table 1: Educational Markets' Characterization

	mean	std. dev.	min	max
students	2,876	1,493	1,025	7,116
% low-income students	54	12	17	71
schools	77	36	27	151
public schools	37	20	8	76
private-voucher schools	35	23	10	105
private-non-voucher schools	4	4	0	14
% private-voucher schools in targeted program	66	18	28	89

Notes: Summary statistics for all 27 geographic educational markets included in the empirical analysis.

Student enrollment in the different types of schools follows some interesting patterns. Table 2 summarizes enrollment, top-up fees, and test scores for schools in my final sample. The vast majority (95%) of students attend a subsidized school, either public or private-voucher. Furthermore, enrollment decisions for low income students are clearly inclined towards schools that charge them no fees, where half of these students attend a public school, and about 39% attend a private-voucher school that is part of the targeted voucher program. This pattern reflects the budget constraint that low income families face when choosing schools, and that motivated the

[18]Figure 5 in Appendix C presents an example of an educational market created with the geocoded data.
[19]See Neilson (2020) for a similar sample selection criterium.

introduction of the targeted voucher. High income students' choices are more dispersed among the different types of schools, where less than a third of these students attend a public zero-fee school, and 11% attend a high fee private-non-voucher school.

Table 2: School Characteristics, by School-type

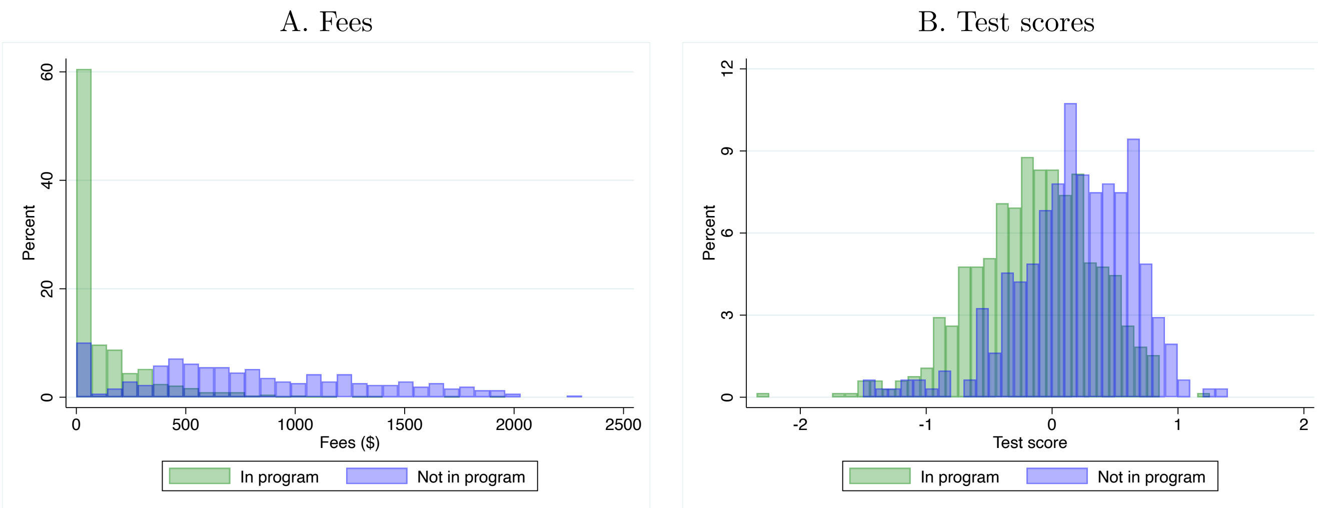
school-type:	public	private-voucher		private-non-voucher
in targeted voucher program:	yes	yes	no	no
enrollment (%)	41	35	19	6
enrollment - low income (%)	50	39	11	1
enrollment - high income (%)	30	30	28	11
top-up fees ($)	0	150	869	6,687
test scores (s.d.)	-0.26	-0.01	0.28	0.78

Notes: Characteristics of primary schools in the analysis sample, depending on whether the school is public, private-voucher, or private-non-voucher, and on whether it participates in the targeted voucher program. Fees are in real 2013 prices, and were transformed from Ch$ to US$ according to the exchange rate as of March 1, 2013 (472.96 Ch$/US$). Test scores are the school average of math and verbal exams and are standardized to have mean zero and standard deviation one at the student level.

Private-voucher school selection into the targeted voucher program is also interesting. It is mostly low-fee, low-test scores schools that decide to join the program, as can be observed in Table 2 and Figure 2. Such pattern reflects the very likely profit seeking motive of private-voucher schools, as was discussed in Section 2. It also suggests that many high-test scores schools decided to stay out of the program, which raises doubts on the capability of the program to make high achieving schools more easily available to low income students.[20]

[20] A similar school sorting pattern was recently observed in the Louisiana Scholarship program, the Louisiana state-funded targeted voucher program studied in Abdulkadiroglu et al. (2018).

Figure 2: Private-Voucher Schools' Fees and Test Scores Distributions



Notes: Fee levels are in real 2013 prices, and were transformed from Ch$ to US$ according to the exchange rate as of March 1, 2013 (472.96 Ch$/US$). Test scores are the school average of math and verbal exams among students, and are in standard deviation units.

5 Estimation and Identification

I estimate the model's parameters sequentially. First, in three steps, I obtain the school popularity and taste heterogeneity parameters in the preferences model, a measure of school value added in auxiliary test scores equations, and then link these two sets of parameters to finalize preferences estimation. Then, given demand parameters, I estimate the parameters that enter schools' marginal cost and nonprofit motive/fixed cost of participating in the targeted voucher program.

5.1 Demand

I proceed in three steps. First, I estimate a measure of each school popularity and the parameters of heterogeneity in students' preferences. Then, I estimate test scores equations, using a selection-on-observables approach, very similar to the work in Neilson (2013), Ferreyra and Kosenok (2018), Allende (2019), Allende et al. (2019), and Singleton (2019). Finally, I link school value added and other characteristics to school popularity estimates to finalize preferences for schools estimation.

5.1.1 Preferences for Schools

I use Maximum Likelihood to estimate preference for proximity, taste heterogeneity, and mean utilities or school popularity. Heterogeneity in preferences is captured by a set of students' observable demographic characteristics, which in practice are mother's educational level dummies. Mean utilities vary at the student's socioeconomic status (low/high income), and absorb the remaining preference components from the indirect utility function,

$$\delta_j^\zeta = \beta_1^\zeta p_j^\zeta + \beta_3^{\zeta\prime} X_j^\beta + \beta_4^\zeta q_j + \xi_j^\zeta$$

The corresponding log-likelihood function is:

$$LL(\beta) = \sum_i \sum_j e_{ij} \ln \left(\frac{\exp\left((\beta_{1i}^\zeta - \beta_1^\zeta) p_j^\zeta + \beta_2^\zeta d_{ij} + (\beta_{3i}^\zeta - \beta_3^\zeta) X_j^\beta + \delta_j^\zeta \right)}{\sum_k \exp\left((\beta_{1i}^\zeta - \beta_1^\zeta) p_k^\zeta + \beta_2^\zeta d_{ik} + (\beta_{3i}^\zeta - \beta_3^\zeta) X_k^\beta + \delta_k^\zeta \right)} \right),$$

where e_{ij} is the choice indicator, i.e. $e_{ij} = 1$ indicates that student i attends school j.

5.1.2 School Value Added

To estimate the measure of school value added, I follow Neilson (2013), Ferreyra and Kosenok (2018), Allende (2019), Allende et al. (2019), and Singleton (2019), and estimate the parameters that enter an auxiliary test scores equation using an offline selection-on-observables and linear-in-the-parameters model. In practice, I run the following OLS regression,

$$Y_{ij} \quad = \quad W_i \gamma + q_j + \epsilon_{ij},$$

where I include a large set of observables in W_i to capture as much variation as possible, and therefore minimize the inconsistency in the estimates. I estimate the test scores equation market by market.

5.1.3 Linking Mean Utilities to School Value Added and Other Characteristics

I use the estimated $\hat{\delta}_j^\zeta$ terms from the first step, as well as the $\hat{q}_j$ estimated school value added from the test scores regression to estimate the remaining mean preference parameters in a linear regression of the form:

$$\hat{\delta}_j^\zeta \quad = \quad \beta_1^\zeta p_j^\zeta + \beta_3^{\zeta\prime} X_j^\beta + \beta_4^\zeta q_j + \xi_j^\zeta. \tag{9}$$

As is usual in demand models, I assume that X_j^β is uncorrelated with ξ_j^ζ. However, I allow

$p_j^\zeta = \tau_j(1-D_i)p_j^1 + (1-\tau_j)p_j^0$ and $\hat{q}_j$ to be endogenous (Cuesta et al., 2020; Neilson, 2020). I thus estimate equation (9) by 2SLS using the following instruments: the average per student universal voucher received by the school, other per student non-voucher subsidies (e.g. teacher incentives, rurality bonus), the share of low income students in the school's census block, and whether the school's neighborhood is considered vulnerable. These instruments are motivated by schools' first order conditions for price, and follow well accepted instruments used in similar contexts (Gazmuri, 2015; Neilson, 2020).

Identification is ensured as long as the variables used to instrument for top-up fees and school value added are valid instruments, in the sense that they are correlated with the endogenous variables, but not with the preferences shock. Subsidies are expected to affect fees by affecting (reducing) the marginal cost of educating a student, as is shown in schools' best responses in Section 3.2. Subsidies are also expected to affect school effectiveness to increase test scores by changing the resources available to schools to invest in educational inputs that determine school effectiveness. At the same time, it is unlikely that subsidies received by schools enter families' preferences for schools, or even that they are observed by families. How vulnerable is the neighborhood where the school is located, or the share of low income families that live close to the school is expected to affect the school's decision to participate in the targeted voucher program, as was argued in Section 2 and suggested by Figure 1, which in turn affects fees via the targeted voucher regulation and affects total resources received by the school. On the other hand, controlling for proximity to schools, families' preferences for schools are unlikely to include the degree of vulnerability of the schools' neighborhood.

5.2 Supply

I parameterize schools' costs structure as follows,

$$\begin{aligned}
c_j^{1,H} &= X_j^\omega \omega_{c^{1,H}} + u_j^{c^{1,H}} \\
c_j^{1,L} &= \omega_{c^{1,L}} c_j^{1,H} \\
c_j^0 &= X_j^\omega \omega_{c^0} + u_j^{c^0} \\
\kappa_j &= Z_j^\omega \omega_\kappa + u_j^\kappa,
\end{aligned}$$

where X_j^ω and Z_j^ω are vectors of observable (to the econometrician and all players) variables affecting marginal costs and nonprofit motives/fixed costs, respectively. The terms $\omega_{c^{1,H}}$, $\omega_{c^{1,L}}$, ω_{c^0} and ω_κ are parameters to be estimated. In particular, $\omega_{c^{1,L}}$ is a scalar denoting the relative weight between $c_j^{1,L}$ and $c_j^{1,H}$. The idiosyncratic error terms, $u_j^{c^{1,H}}$, $u_j^{c^0}$ and u_j^κ, are independent and normally distributed with mean zero and variance terms $\sigma^2_{c^{1,H}}$, $\sigma^2_{c^0}$ and σ^2_κ, respectively. The variance terms are also parameters to be estimated.

Conditional on observables X_j^ω and Z_j^ω, all costs elements are statistically independent, both within and across schools, i.e. $c_j^{1,H} \perp\!\!\!\perp c_{j'}^{1,L} \perp\!\!\!\perp c_{j''}^0 \perp\!\!\!\perp \kappa_{j'''} \mid X^\omega, Z^\omega$ for any j, j', j'', j''', where $\perp\!\!\!\perp$ denotes statistical independence.

I combine demand estimates and schools' optimality conditions (2)–(8) to recover the parameters governing schools' costs structure. Let $\bar{s}_j(\bar{s}_{-j}) \equiv \int_{T_j} s_j(\bar{s}_{-j}; t_j)dF(t_j)$ be school j's fully cursed strategy, where T_j is the space of school j's types. Thus,[21]

$$\begin{aligned}
\bar{p}_j^1(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j}) \quad = \quad & \left[1 - \Phi\left(\frac{-\left(X_j^\omega \omega_{c^{1,H}} - v^u - \frac{\sum_i (1-D_i) P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\sum_i (1-D_i)\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\partial p_j}}\right)}{\sigma_{c^{1,H}}}\right)\right] \\
& \left[X_j^\omega \omega_{c^{1,H}} - v^u - \frac{\sum_i (1-D_i) P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\sum_i (1-D_i)\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\partial p_j}}\right. \\
& \left. + \sigma_{c^{1,H}} \lambda \left(\frac{-\left(X_j^\omega \omega_{c^{1,H}} - v^u - \frac{\sum_i (1-D_i) P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\sum_i (1-D_i)\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\partial p_j}}\right)}{\sigma_{c^{1,H}}}\right)\right], \qquad (10)
\end{aligned}$$

where $\lambda(\nu) = \frac{\phi(\nu)}{1-\Phi(\nu)}$ is the inverse Mill's ratio, and $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and probability distribution, respectively. Note that,

$$P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j}) \quad = \quad \frac{e^{V_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}}{\sum_k e^{V_{ik}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}},$$

where, $V_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j}) = \beta_{1i}\left(\bar{\tau}_j(1-D_i)\bar{p}_j^1 + (1-\bar{\tau}_j)\bar{p}_j^0\right) + \beta_2^\zeta d_{ij} + \beta_{3i}' X_j + \beta_4^\zeta q_j + \xi_j^\zeta$. Also,

$$\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})}{\partial p_j} \quad = \quad \beta_{1i} P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j}) \left(1 - P_{ij}(\bar{s}_j(\bar{s}_{-j}), \bar{s}_{-j})\right).$$

[21]For ease of notation, I let implicit the conditionality on X_j^ω and Z_j^ω across the different pieces of the supply side model.

Similarly,

$$
\begin{aligned}
\bar{p}_j^0(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}) \quad = \quad & \left[1-\Phi\left(\frac{-\left(X_j^\omega\omega_{c^0}-v^u-\frac{\sum_i P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\sum_i\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\partial p_j}}\right)}{\sigma_{c^0}}\right)\right] \\
& \left[X_j^\omega\omega_{c^0}-v^u-\frac{\sum_i P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\sum_i\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\partial p_j}}\right. \\
& \left.+\sigma_{c^0}\lambda\left(\frac{-\left(X_j^\omega\omega_{c^0}-v^u-\frac{\sum_i P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\sum_i\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\partial p_j}}\right)}{\sigma_{c^0}}\right)\right]. \qquad (11)
\end{aligned}
$$

Finally,

$$
\bar{\tau}_j(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}) \quad = \quad \Phi\left(\frac{\Pi_j^1\left(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}\right)-\Pi_j^0\left(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}\right)+Z_j^\omega\omega_\kappa}{\sigma_\kappa}\right), \qquad (12)
$$

where,

$$
\begin{aligned}
\Pi_j^1\left(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}\right) \quad = \quad & \left(\bar{p}_j^1(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})+v^u-X_j^\omega\omega_{c^{1,H}}\right)\sum_i(1-D_i)P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}) \\
& +\left(v^u+v^t-\omega_{c^{1,L}}(X_j^\omega\omega_{c^{1,H}})\right)\sum_i D_iP_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}), \\
\Pi_j^0\left(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}\right) \quad = \quad & \left(\bar{p}_j^0(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})+v^u-X_j^\omega\omega_{c^0}\right)\sum_i P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}).
\end{aligned}
$$

Notice that, since X_j^ω and Z_j^ω are observed by everyone in the game, school j's cost structure is private information only because the stochastic part of the cost terms—i.e. $\left(u_j^{c^{1,H}},u_j^{c^0},u_j^\kappa\right)$—is unobserved by j's competitors. That is, school j's type is given by its realization of the idiosyncratic error terms.

Identification proceeds as follows. $\bar{p}_j^1(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})$ is the conditional mean of a normal censored (from below at zero) variable. Normality and variation from the observables ensures the identification of $\omega_{c^{1,H}}$ and $\sigma_{c^{1,H}}$. Moreover, the model is overidentified, because theory imposes the coefficient accompanying $\frac{\sum_i(1-D_i)P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\sum_i(1-D_i)\frac{\partial P_{ij}(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})}{\partial p_j}}$ to be equal to one. A similar argument identifies ω_{c^0} and σ_{c^0}. From the probit model for $\bar{\tau}_j(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j})$, variation in Z_j^ω identifies $\frac{\omega_\kappa}{\sigma_\kappa}$. The coefficients accompanying $\Pi_j^1\left(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}\right)$ and $\Pi_j^0\left(\bar{s}_j(\bar{s}_{-j}),\bar{s}_{-j}\right)$ are constrained to be one by the theory, which allows the identification of two additional parameters, σ_κ and $\omega_{c^{1,L}}$.

I utilize a Generalized Method of Moments (GMM) routine to estimate the parameters in the supply. I combine GMM with a Mathematical Programming with Equilibrium Constraints (MPEC; Dubé et al., 2012; Su and Judd, 2012) approach, that minimizes the objective function of moment conditions subject to markets' equilibria, i.e. fixed point of schools' best responses in each market.

I choose the following sample moments to construct the GMM objective function,

$$
\begin{aligned}
g_{p^1} &= \frac{1}{\sum_{m=1}^{M} J_m} \sum_{m=1}^{M} \sum_{j=1}^{J_m} \tau_{j,m} X_j^{\omega\prime} \left(p_{j,m} - \bar{p}_{j,m}^1(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m})\right), && (13)\\
g_{p^0} &= \frac{1}{\sum_{m=1}^{M} J_m} \sum_{m=1}^{M} \sum_{j=1}^{J_m} (1-\tau_{j,m}) X_j^{\omega\prime} \left(p_{j,m} - \bar{p}_{j,m}^0(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m})\right), && (14)\\
g_{p} &= \frac{1}{\sum_{m=1}^{M} J_m} \sum_{m=1}^{M} \sum_{j=1}^{J_m} X_j^{\omega\prime} (p_{j,m} && (15)\\
&\quad -(\tau_{j,m}\bar{p}_{j,m}^1(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m}) + (1-\tau_{j,m})\bar{p}_{j,m}^0(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m}))), && (16)\\
g_{\tau} &= \frac{1}{\sum_{m=1}^{M} J_m} \sum_{m=1}^{M} \sum_{j=1}^{J_m} Z_j^{\omega\prime} (\tau_{j,m} - \bar{\tau}_{j,m}(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m})), && (17)\\
g_{p^1_m} &= \frac{1}{M} \sum_{m=1}^{M} \left(\frac{1}{J_m} \sum_{j=1}^{J_m} \tau_{j,m} \left(p_{j,m} - \bar{p}_{j,m}^1(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m})\right) \right), && (18)\\
g_{p^0_m} &= \frac{1}{M} \sum_{m=1}^{M} \left(\frac{1}{J_m} \sum_{j=1}^{J_m} (1-\tau_{j,m}) \left(p_{j,m} - \bar{p}_{j,m}^0(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m})\right) \right), && (19)\\
g_{p_m} &= \frac{1}{M} \sum_{m=1}^{M} \left(\frac{1}{J_m} \sum_{j=1}^{J_m} (p_{j,m} - \right. && (20)\\
&\quad \left. (\tau_{j,m}\bar{p}_{j,m}^1(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m}) + (1-\tau_{j,m})\bar{p}_{j,m}^0(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m}))) \right), && (21)\\
g_{\tau_m} &= \frac{1}{M} \sum_{m=1}^{M} \left(\frac{1}{J_m} \sum_{j=1}^{J_m} (\tau_{j,m} - \bar{\tau}_{j,m}(\bar{s}_{j,m}(\bar{s}_{-j,m}), \bar{s}_{-j,m})) \right), && (22)
\end{aligned}
$$

where $p_{j,m}$ and $\tau_{j,m}$ are the actual prices and participation decisions observed in the data, and J_m is the total number of private-voucher schools in market m. Sample moments (13)–(17) are implied by the orthogonality conditions between the error terms and the observables X^ω and Z^ω in the equations for $p_{j,m}^1$, $p_{j,m}^0$, and $\tau_{j,m}$. These moments are sufficient to consistently estimate all supply parameters. Nonetheless, I add market-level moments (18)–(22) to improve the fit of the model in each market. These moments minimize the difference between actual market-averages and market-averages implied by the model for $p_{j,m}^1$, $p_{j,m}^0$, and $\tau_{j,m}$.

Vertically stack sample moments in $g(\omega, \bar{s}) = (g_{p^1}, g_{p^0}, g_p, g_\tau, g_{p^1_m}, g_{p^0_m}, g_{p_m}, g_{\tau_m})$. Also. let

$$W = diag\left(\left(\frac{1}{\sum_{m=1}^{M} J_m} X^{\omega\prime} X^{\omega}\right)^{-1}, \left(\frac{1}{\sum_{m=1}^{M} J_m} X^{\omega\prime} X^{\omega}\right)^{-1}, \left(\frac{1}{\sum_{m=1}^{M} J_m} X^{\omega\prime} X^{\omega}\right)^{-1}, \left(\frac{1}{\sum_{m=1}^{M} J_m} Z^{\omega\prime} Z^{\omega}\right)^{-1}, I_4\right),$$

where $diag(v)$ denotes a matrix with v in its diagonal and zeros elsewhere, and I_4 is the identity matrix of dimension 4. Finally, let $h(\omega, \bar{s})$ be the $3\sum_{m=1}^{J_m} \times 1$ vector of equilibrium conditions (10)–(12).

The GMM-MPEC problem is,

$$\min_{\omega, \bar{s}} \quad g(\omega, \bar{s})' W g(\omega, \bar{s}),$$

subject to,

$$h(\omega, \bar{s}).$$

The MPEC algorithm simultaneously searches over parameters ω and endogenous variables $\bar{s}$, to minimize the objective function subject to equilibrium conditions to be satisfied. As in many industrial organization applications, this problem is sparse, since the equilibrium conditions need to hold for each market separately. The sparsity of the problem allows to find a solution relatively quickly compared to other approaches such as Nested Fixed Point algorithms, that require solving for equilibria at each guess of the parameters (Dubé et al., 2012; Su and Judd, 2012). In practice, I implement the GMM-MPEC routine in MATLAB/TOMLAB using the professional solver KNITRO. I provide the automatic differentiated Jacobian and (approximated) Hessian using TOMLAB's TomSym package.

6 Results and Analysis

6.1 Estimates

6.1.1 School Value Added

I perform estimation imposing a selection-on-observables assumption (Neilson, 2013; Ferreyra and Kosenok, 2018; Allende, 2019; Allende et al., 2019; Singleton, 2019), where I run a linear regression of test scores on a large set of observable characteristics and school indicators, market by market. I use the student-level average of 4th grade national standardized scores in verbal and math exams. The set of covariates I include in the regressions are gender, low income status, grade repetition, mother's education, father's education, household income, having attended pre-K, having attended kindergarten, computer availability at home, internet availability at home, indigenous status, number of books at home, and class attendance. For brevity, I do not present each market's estimates, but they are available upon request.

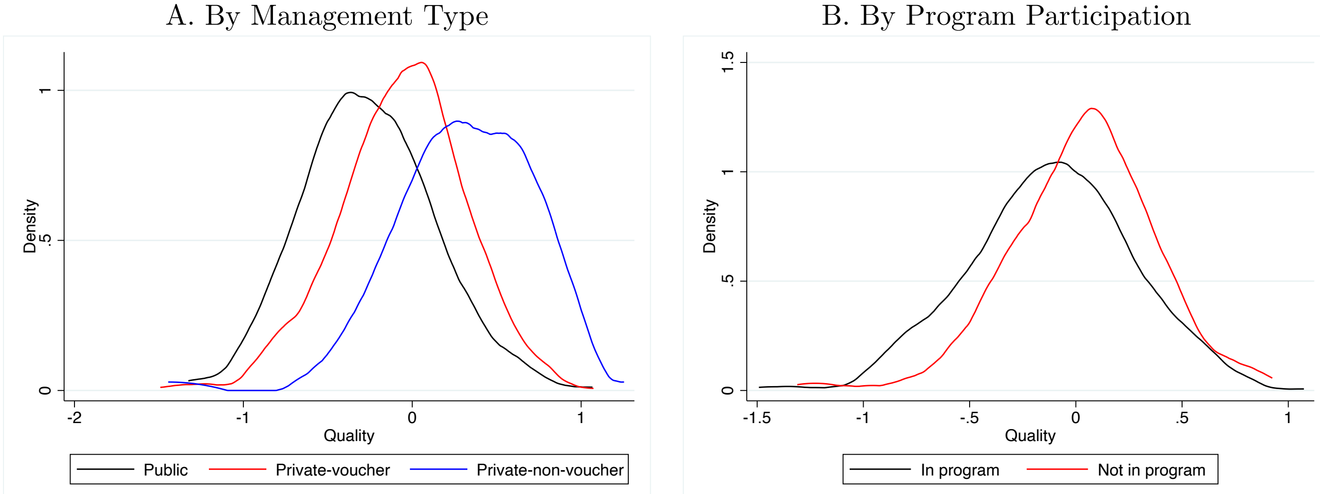
I summarize schools' value added or quality estimates in density plots. Interesting patterns emerge when the quality distribution is split by school's management type. Panel A in Figure 3 shows that the quality distribution of private-non-voucher schools is shifted to the right of the quality distribution of private-voucher schools, which in turn is shifted to the right of public schools' corresponding distribution. The differences in the distributions are not hard to visualize, denoting the clear ordering (in means) of school value added among management types. This finding is in line with the evidence in Neilson (2020), that shows very similar differences between public, private-voucher, and private-non-voucher schools' distributions of estimated quality.

Also interesting is the difference in value added distributions among private-voucher schools that participate in the targeted voucher program and private-voucher schools that do not participate. Panel B in Figure 3 suggests that the program attracted many low quality schools. Although there exists an overlap between the two groups' distributions, the majority of high quality schools decided to stay out of the program.

Figure 3: Distribution of School Value Added by School Type

A. By Management Type

B. By Program Participation



Notes: Panel A displays nonparametric estimates of the distribution of school value added, by management type of the school. Panel B displays nonparametric estimates of the distribution of school value added for private-voucher schools, by participation in the targeted program status.

6.1.2 Preferences

I present estimates for families' preferences in Table 3, separated by low and high income. Also, within family income group, preferences for most school characteristics are allowed to vary by mother's educational level, where the omitted category is "more than high school". Low income families are more price sensitive than higher income families, which is consistent with the evidence in related studies (Neilson, 2020). There is also important heterogeneity within each family income group, where families with less educated mothers are more responsive to price changes than families with educated mothers. Both low and high income families value higher levels of school effectiveness, although estimates suggest that quality is somewhat more important for high income families. In general, families prefer schools that are secular, private, and that offer a full day shift, with important heterogeneity in the preferences for these attributes among the different groups. Finally, proximity to school is an attribute both low and high income families consider preferable when choosing primary schools.

Table 3: Estimates for Demand Model

	low income		high income	
	coef.	std. err.	coef.	std. err.
fees	-0.941	0.188	-0.087	0.161
fees × mother's education: less than high school	-1.777	0.049	-1.563	0.026
fees × mother's education: high school	-0.592	0.030	-0.560	0.011
fees × mother's education: missing	-0.324	0.038	-0.041	0.012
school value added	0.390	0.098	0.753	0.083
secular	0.142	0.072	0.179	0.059
secular × mother's education: less than high school	0.050	0.036	-0.046	0.037
secular × mother's education: high school	0.052	0.035	-0.080	0.029
secular × mother's education: missing	0.175	0.051	0.020	0.050
public	-1.979	0.182	-1.689	0.165
public × mother's education: less than high school	1.158	0.040	0.974	0.043
public × mother's education: high school	0.500	0.039	0.513	0.036
public × mother's education: missing	1.238	0.056	1.495	0.060
full day shift	0.588	0.157	0.345	0.126
full day shift × mother's education: less than high school	0.169	0.052	-0.020	0.054
full day shift × mother's education: high school	0.013	0.050	-0.175	0.042
full day shift × mother's education: missing	-0.045	0.075	-0.266	0.079
distance to school	-0.009	0.000	-0.072	0.000
no. of students	77,656			

Notes: Results from maximum likelihood estimation of distance and preference heterogeneity by mother's education, and from 2SLS estimation of mean preference parameters. Omitted mother's level of education category is "more than high school". Fee amounts are in real $1,000 for the year 2013, and were transformed from Ch$ to US$ according to the exchange rate as of March 1, 2013 (472.96 Ch$/US$). Distance to school is in 100 meters. Estimations include market fixed effects, whose corresponding estimates are omitted.

6.1.3 School Costs

I consistently estimate schools' cost structure using a random sample of 200 students per market. Estimates for schools' marginal costs and participation cost/nonprofit motive are presented in Table 4. Estimated coefficients for the marginal cost functions, c_j^0 and $c_j^{1,H}$, confirm my modeling decision of allowing the marginal cost function to vary by school's participation in the program decision, as is the case in a typical Roy model. Most coefficients are in line with economic intuition and evidence suggesting that educating students from vulnerable family backgrounds is more costly than educating students from non-vulnerable families, that subsidies (partly) alleviate such costs, and that producing higher levels of quality of education raises costs (Fontaine and Urzúa, 2018). Interestingly, results imply that program participation may be related to some efficiency gains. This finding is especially true for the production of school value added, where participation is associated with a sharp decrease in that cost—the estimated coefficient accompanying the school value added variable goes from positive when not in the program (c_j^0), to negative in the in-the-program regime ($c_j^{1,H}$). The estimated coefficient for $\omega_{c^{1,L}}$ implies that the marginal cost of educating a low income student is somewhat larger than the marginal cost of educating a high income student. Once more, the estimation results support my modeling choice for the cost structure of schools.

The estimates for the κ_j function show how observable shifters determine schools' nonprofit motive/participation cost associated to joining the program and, consequently, schools' probability of entering the targeted voucher program (equation (12)). Schools that are more attracted to join the program are schools that do not offer full day shift, are surrounded by low income students in less developed neighborhoods, receive a relatively high amount of subsidies other than the vouchers, are relatively small, and have low levels of school value added. The latter result is in line with the evidence in Abdulkadiroglu et al. (2018), that show a negative relationship between school participation in the Louisiana Scholarship Program and the quality of education of the school.

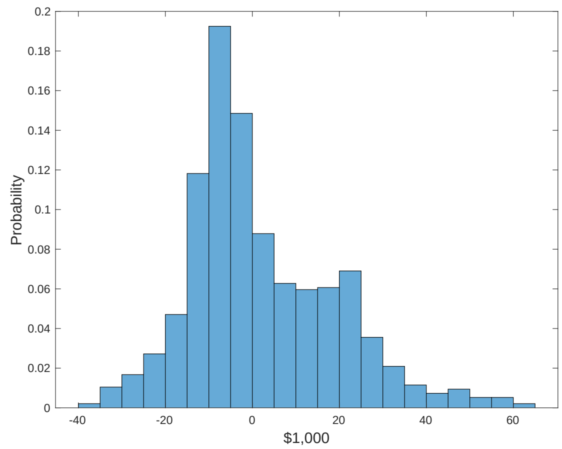
Table 4: Estimates for Supply Model

	coef.	std. err.	coef.	std. err.
	$\mathbf{c_j^0}$		$\mathbf{c_j^{1,H}}$	
full day shift	0.407	0.231	-0.064	0.008
low income students in municipality (%)	1.158	0.824	0.322	0.277
neighborhood's vulnerability index	3.053	2.659	-7.060	2.667
non-voucher subsidies	-0.344	0.242	0.031	0.018
school value added	0.119	0.011	-0.049	0.039
constant	-3.290	0.293	2.424	0.806
$\ln\sigma$	1.076	0.190	-8.306	2.273
	$\mathbf{c_j^{1,L}}$			
$\omega_{c^{1,L}}$	1.326	0.907		
	$\boldsymbol{\kappa_j}$			
full day shift	-25.762	9.991		
low income students in municipality (%)	7.745	3.364		
neighborhood's vulnerability index	-9.966	1.800		
non-voucher subsidies	8.389	7.041		
no. of classes	-1.311	0.275		
school value added	-13.505	7.238		
constant	18.697	3.059		
$\ln\sigma$	-6.796	5.332		
no. of private-voucher schools		956		

Notes: All supply parameters were estimated using a GMM-MPEC procedure. Standard errors were computed using a parametric bootstrap, where 50 synthetic data sets were generated using the estimated cost structure functions and markets' equilibria (Efron and Tibshirani, 1993; Su and Judd, 2012). Costs are in real $1,000 for the year 2013, and were transformed from Ch$ to US$ according to the exchange rate as of March 1, 2013 (472.96 Ch$/US$). Estimations include market fixed effects, whose corresponding estimates are omitted.

Figure 4 plots the estimated distribution for the nonprofit motive/fixed cost of participating in the targeted voucher program, κ_j. Heterogeneity among schools is important, where for some the κ_j term is negative, implying that the costs associated to program participation are larger than the benefits, while for others κ_j is positive, implying that the nonprofit motive that induces participation outweighs any fixed cost schools may incur to be part of the program.

Figure 4: Estimated Participation Costs/Nonprofit Motive



Notes: Estimated nonprofit motive/participation costs are in real $1,000 for the year 2013, and were transformed from Ch$ to US$ according to the exchange rate as of March 1, 2013 (472.96 Ch$/US$).

6.2 Policy Analysis and Counterfactuals

I use the model and its estimated parameters to study the equilibrium consequences of a variety of counterfactual policy scenarios. I focus on the strategic choices schools make, and on how these choices correlate with some exogenous school characteristics, e.g quality. My aim is to show that a model that endogeneizes both participation and fee decisions is necessary to fully understand the consequences of policies, and to study alternative policy designs whose results may be better aligned with policymakers' objectives.

I begin by studying the importance of accounting for both extensive and intensive margins responses of schools to voucher policies. I compare the predictions of this paper's model with the predictions of models that allow schools to respond in only one margin, either program participation or fees. Table 5 displays the results of the counterfactual exercises. Five scenarios are simulated, one where no targeted voucher is allowed and thus schools receive the universal voucher only, and four others in which the targeted subsidy is in place and is set to either 25%, 50%, 75%, or 100% its actual value ($862 in 2013). The universal voucher is fixed to its actual value in all five counterfactuals ($1,305).

Panel A in Table 5 examines the program participation responses of private-voucher schools as predicted by this paper's model (full model), and by a restricted model that allows schools to endogenously select only their participation action. In the restricted model, I fix schools' top-up fees to their actual values. In both models, schools' participation in the program increases as the targeted voucher rises. However, the restricted model predicts a more attenuated response than the full model. By not allowing schools to also respond in fees, the restricted model does not capture the direct effect in fees that a change in the targeted voucher has on schools, as well as the indirect (best response) effect in fees induced by school's competitors own (participation and fees) responses to changes in the subsidy. Importantly, the mispredictions of the restricted model are sizable.

Panels B and C in Table 5 display the top-up fee responses of private-voucher schools as predicted by the full model, and by a restricted model that allows schools to endogenously select only their top-up fees. In the restricted model, I fix schools' participation action to their actual values. I distinguish between fees charged to high income students (panel B), and fees charged to low income students (panel C), where the latter include the mandated zero fees for schools that opt into the program. When the targeted voucher is not allowed (first column), both models predict the same fees, as expected. Now, when the targeted voucher is turned on, the restricted model predicts a constant fee level regardless of the value of the subsidy. The reason for this lack of sensitivity of top-up fess to subsidy changes is that the restricted model fails at capturing the direct and best response effects of the targeted voucher on fees, both of which are induced by schools' participation decisions (equations (2)-(7)). In contrast, the full model predicts a monotonic decrease in the fees charged to both high and low income students as the subsidy level rises. Notice, too, that even when the targeted voucher is set to its actual value (100%), the restricted model underpredicts schools' top-up fees. This result is, again, explained by the restricted model not accounting for the effect that fees have on participation, and viceversa. In the restricted model, when a school sets a lower fee level, it only internalizes other schools' best-responses on their own fees, and ignores the competition effect that a lower fee level has on other schools' participation choices.

All in all, a higher targeted voucher induces greater program participation as well as lower levels of top-up fees. More importantly, a correct prediction of the consequences of policies necessitates accounting for both program participation and fees decisions.

Table 5: Models' Predictions Under Different Targeted Voucher Amounts

	voucher amount (% actual)				
	0	25	50	75	100
A. Participation (share)					
full model	0.00	0.48	0.53	0.59	0.64
restricted model (participation)	0.00	0.36	0.40	0.44	0.48
B. Fees for high income ($)					
full model	591	402	395	369	373
restricted model (fees)	591	349	349	349	349
C. Fees for low income ($)					
full model	591	290	264	238	214
restricted model (fees)	591	181	181	181	181

Notes: Private-voucher schools' program participation and top-up fees responses under different amounts of the targeted voucher, and as predicted by this paper's model and models that restrict schools to respond only in one margin, either program participation or top-up fees. Fee levels are in real prices as of 2013, and were transformed from Ch$ to US$ according to the exchange rate as of March 1, 2013 (472.96 Ch$/US$).

Next, I study the sorting on school value added that is induced by the current targeted voucher program. I simulate two counterfactual scenarios, one without the program, and another with the actual (as of 2013) level of the targeted subsidy. In both cases, the universal voucher amount is fixed at its actual level ($1,305). Table 6 shows the corresponding model's predictions. In the counterfactual without the targeted program, the average level of private-voucher schools' value added is 0.07 s.d. When the targeted voucher program is in place, the simulations show a striking sorting on school value added, where participating schools' average value added, -0.04 s.d., is well below the corresponding value for non-participants, 0.27 s.d. This result is in line with the suggesting raw data evidence in Figure 2, and with the estimation results in Figure 3 and Table 4.

Table 6: Sorting on School Value Added

	no program	actual program
participants	–	-0.04
non-participants	0.07	0.27

Notes: School value added is in standard deviations of the test scores distribution. Each cell represents the average school value added of private-voucher schools that belong to the group defined by the corresponding row and column headers, e.g. participating schools under the actual targeted program.

The results presented in Table 6 on the sorting on school value added motivate the study of alternative programs that may attract higher quality schools to participate in the targeted program. One direct way of attracting high value added schools is to offer them larger subsidy amounts for participating in the program. I therefore study three alternative, similarly costly, voucher policies, and examine the implied sorting on school value added, as well as participation and top-up fee strategies in equilibrium. One counterfactual policy gives 150% the actual targeted voucher amount to private-voucher schools with a value added in the top half of the value added distribution of private-voucher schools, and gives 50% the actual targeted voucher to the rest of private-voucher schools. Another counterfactual policy gives 200% the actual targeted voucher amount to private-voucher schools with a value added in the top half of the value added distribution, and gives no targeted voucher to the rest of private-voucher schools (while still allowing these schools to join the program). A final counterfactual policy assumes away the targeted voucher program, and increases the universal voucher to 150% its actual value. I additionally compare the implications of these three counterfactuals to the actual program.

Table 7 displays the results of the model's simulations for each counterfactual policy. For reference, the first column presents the equilibrium consequences of the actual program, which includes a universal voucher of $1,305 and a targeted voucher of $862. Column (2) presents the results of the alternative policy that rewards high quality schools (i.e. schools in the top half of the value added distribution) with 150% the actual targeted voucher, and gives 50% the actual targeted voucher to the rest of private-voucher schools. Column (3) presents the results of the policy that gives 200% the actual voucher to high quality schools and zero targeted subsidy to the rest of private-voucher schools. Finally, column (4) shows the equilibrium consequences of the policy that includes only a universal voucher of 150% the actual amount for all schools.

Panel A shows that both alternative policies that include a targeted voucher (columns (2) and (3)) attract a very similar share of private-voucher schools to join the targeted program than the actual policy (column (1)). There are, however, important differences in the value added of the schools that choose to participate among the policies. In line with private-voucher schools' (net) profit seeking behavior, increasing the targeted voucher for high quality schools incentivizes more of these schools to opt in, while reducing the targeted subsidy for low quality schools keeps some of these schools out of the program. The combination of these two effects results in a higher quality pool of private-voucher schools that charge zero fees to low income students, and that consequently are more likely to be chosen by vulnerable students. Panel B shows that paying high quality schools 150% the actual targeted voucher increases the average value added of participant schools from -0.04 s.d. (column (1)) to 0.02 s.d. (column (2)). Furthermore, doubling the actual targeted voucher for high quality schools implies an even higher quality pool of participants, with an average value added of 0.08 s.d. (column (3)). In fact, this alternative policy is the only one where participants outperform non-participants in school value added. Panel C shows that while the alternative policies that include a targeted voucher (columns (2) and (3)) imply higher fees for high income students than the actual policy (column (1)), they also result in lower average fees for low income students. The policy with no targeted voucher and a higher universal voucher (column (4)) results in higher fees than all other policies. Similar to the analysis in Table 5, as this policy shuts down the targeted voucher, it eliminates the best response of schools to others' program participation channel of competition. As a result, schools are less sensitive to subsidy changes. Lastly, panel D shows that all policies are similarly costly, and thus feasible for a fixed budget the size of the actual program.

Table 7: Equilibrium Strategies and Quality Sorting Implications of Alternative Policies

universal (% actual):	100	100	100	150
targeted (% actual):	100	150/50	200/0	–
	(1)	(2)	(3)	(4)
A. Participation (share)				
participation	0.64	0.65	0.65	–
B. School value added (s.d.)				
participants	-0.04	0.02	0.08	–
non-participants	0.27	0.17	0.07	0.07
C. Top-up fees ($)				
high income	373	381	385	470
low income	214	205	199	470
D. Subsidy cost ($ million)				
low income	69	67	67	60
high income	59	58	58	53
all	128	125	125	113

Notes: Counterfactual policy (1) is the actual combination of universal and targeted voucher amounts. Counterfactual policy (2) sets the universal voucher at its actual value, and the targeted voucher at 150% its actual value for private-voucher schools with value added in the top half of the value added distribution and at 50% for the rest of private-voucher schools. Counterfactual policy (3) sets the universal voucher at its actual value, and the targeted voucher at 200% its actual value for private-voucher schools with value added in the top half of the value added distribution and at zero for the rest of private-voucher schools. Counterfactual policy (4) sets the universal value at 150% its actual value, and does not include a targeted voucher.

Summing up, policy simulations show that models that do not allow schools to respond to policies in both extensive and intensive margins may mispredict the equilibrium consequences of policies in a sizable fashion. Simulation exercises also show that the voucher policy that is currently in place in Chile (as of 2013) is not successful in attracting high value added schools to participate in the voucher program that is targeted to economically disadvantaged students. This empirical result has also been found for similar programs in other contexts, such as the Louisiana Scholarship Program (Abdulkadiroglu et al., 2018). The model presented in this paper allows the study of alternative, similarly costly, voucher policies that are able to attract a higher quality pool of schools to participate in the program aimed at low income students.

7 Conclusions

I present an empirical model that allows the estimation of static discrete-continuous games of incomplete information with many players, that are typically found in some industries, such as private school competition in education. The model I develop is motivated by and tailored to the Chilean primary education market, but can easily be adapted to other industries in other contexts.

To circumvent the curse of dimensionality problem inherent to (incomplete information) static games where a large number of players compete in both discrete and continuous strategies, I adapt the *fully cursed* equilibrium concept to my setting (Eyster and Rabin, 2005). In a fully cursed equilibrium, players know that there exists a distribution of types in the population, but ignore the correlation between their opponents' types and actions. Instead, players assume others play the first moment of the types distribution, regardless of their realized type. In large games, such simplifications of standard notions of equilibrium have found empirical support (Kagel and Levin, 1986; Eyster and Rabin, 2005). Related simplifications for dynamic games have also been developed and applied to real world regulation questions (Weintraub et al., 2008; Qi, 2013).

In my application, I show that restricted models that only account for either the intensive or the extensive margins response of schools to vouchers may largely mispredict the equilibrium consequences of policies.

I use my model to study alternative, similarly costly, policies that may attract a higher quality pool of schools to serve economically disadvantaged students than the actual policy in place. My simulation results are informative, and offer some degree of fine tuning among alternative policies, which can prove to be useful to policymakers in search of particular equilibrium results of programs.

My work extends important other studies in the education and other industries (Seim, 2006; Neilson, 2013). At the same time, this paper motivates further developments in the field. Accounting for externalities in the demand side (Ferreyra and Kosenok, 2018; Allende, 2019), endogenizing the choice of school quality (Allende, 2019; Allende et al., 2019), or modeling market entry and exit (Singleton, 2019; Dinerstein and Smith, 2021) are all avenues of future research that can benefit from the work I present here.

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A Discussion on the Supply Side Modeling Assumptions

Profit maximization. I assume all private-voucher schools are profit seekers. This assumption is convenient as it simplifies the choice of the objective function of schools. An alternative is to model schools as being not-for-profit. However, it is not clear what is the objective function of not-for-profit schools. Moreover, the for-profit assumption for Chile's schools is standard and accepted in the literature (Urquiola and Verhoogen, 2009; Allende et al., 2019; Neilson, 2020).
Incomplete information. I assume that schools' costs are private information, and therefore the static game that schools play is one of incomplete information. This is a reasonable assumption as long as costs include both public information components such as teachers' salaries and utilities' bills, and private information components such as schools' intrinsic efficiency and level of bureaucracy. I argue this is the case for the context I study. As supporting evidence, Allende et al. (2019) find that, in Chile, schools' costs are partly determined by the level of skills of the schools' principals, which is usually unobserved to the other schools.
Marginal costs. I allow marginal costs of education to vary by schools' program participation regime (τ_j), and by students' income status (ζ), i.e. $c_j^{\tau_j,\zeta}$. Different costs across regimes allow for the targeted program to have an efficiency effect on schools' production of education. Also, different costs across students' socioeconomic status capture the fact that educating an economically disadvantaged student, who is likely to come from a vulnerable and at risk family, may involve more educational efforts than educating a non-disadvantaged student, who is likely to experience a richer and more stimulating environment at home. Fontaine and Urzúa (2018) discuss this latter phenomenon for the Chilean context. In such respect, I improve on previous studies, both in education markets and in other industries, that restrict costs to be invariant to firms' decisions (and, for the case of education, to students' characteristics).

B Data

Below, I present a detailed description of the data sets used in this paper:[22]

- *Registry of students, 2013.*
  These data provide information on students' gender, date of birth, age, residential address, type and level of education, grade, class, grade repetition status, special education status, and various characteristics of the school of attendance, such as municipality, type of management (public, private-voucher, private-non-voucher), single/double shift schedule, and

[22]These data sets were kindly provided by the Chilean Ministry of Education and Agencia de Calidad de la Educación.

urban status.

- *Registry of schools, 2013.*
  These data provide information on schools' municipality, type of management, urban status, address, tuition, religious orientation and type and level of education offered.

- *Registry of students that are eligible to participate in the targeted voucher program, 2013.*
  These data provide information on the characteristics of students that are eligible to participate in the targeted voucher program. They provide information on students' gender, date of birth, program participation status, level of education, grade, single/double shift schedule, and on the type of management and urban status of the school attended by the student.

- *Registry of schools that participate in the targeted voucher program, 2013.*
  These data provide information on the characteristics of the schools that participate in the targeted voucher program. Information on schools' municipality, type of management, urban status, number of disadvantaged students that are eligible for the targeted voucher subsidy, and number of students that are beneficiary of the targeted voucher is available.

- *National standardized exams (SIMCE) for 4th graders, student-level, 2013*
  These data provide information on students' test scores for three different subjects: verbal, mathematics, and natural sciences.

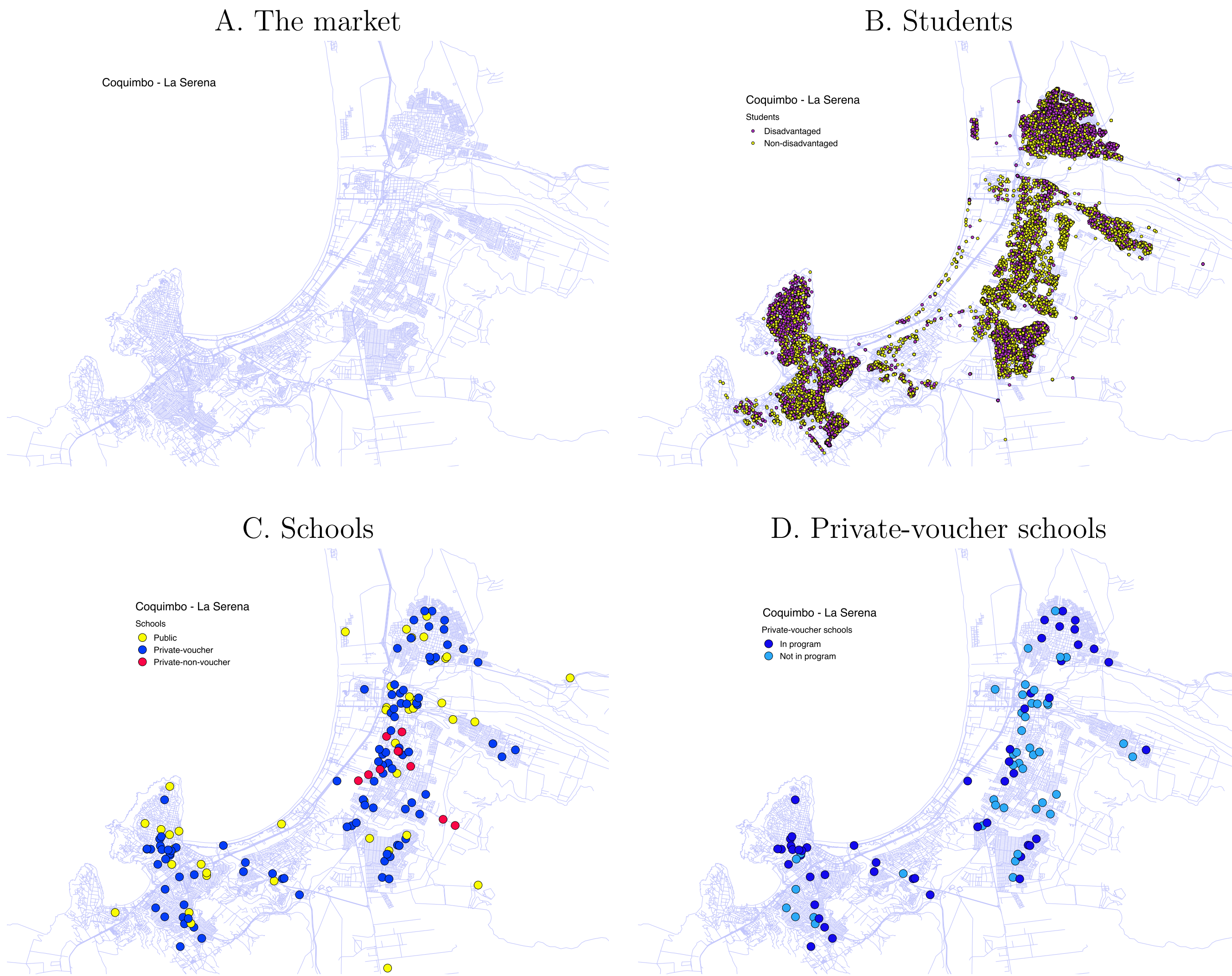
- *4th grade SIMCE's questionnaire to parents and tutors, 2013.*
  These data consist in the responses to a survey that parents and tutors answer during the days when the national standardized tests are taken. The survey is voluntary, though more than 90% of parents respond it every year. It provides information on students' household size, house amenities and time use, total number of books available in the house, household total monthly income, parents and tutors' time use, education, indigenous identification, occupation, health insurance, participation in social programs, reasons for the choice of the school, beliefs on the student's future educational attainment, satisfaction with the school, knowledge of the school's average performance in standardized tests, total monthly expenses related to the student's education other than tuition, and school's admission criteria, tuition and fees.

C A Market Example

Figures 5 presents an example of an educational market created with the geocoded data. The market is formed by the municipalities of Coquimbo and La Serena in Northern Chile. Panel

A displays the streets and roads layout for the market. Panel B displays the spatial distribution of students' homes within the market. It distinguishes between economically disadvantaged (in purple) and non-disadvantaged (in yellow) students. Notice that it is possible to identify neighborhoods with high and low concentrations of disadvantaged students. Panel C displays the spatial distribution of schools within the market, distinguishing between public (in yellow), private-voucher (in blue), and private-non-voucher (in red) schools. Here, we can also identify areas with different concentrations of privately managed schools. Finally, Panel D displays the spatial distribution of private-voucher schools, distinguishing between schools that participate (in blue) and do not participate (in light blue) in the targeted voucher program. Not surprisingly, neighborhoods with high concentrations of disadvantaged students (in Panel B) also present high concentrations of schools that opted to participate in the targeted voucher program. Nonetheless, both types of schools are found in all neighborhoods.

Figure 5: Educational Market: Coquimbo-La Serena



Notes: This figure presents the educational market formed by the municipalities of Coquimbo and La Serena, in Northern Chile.